# Il faut sauver la princesse

1) Référentiel terrestre supposé galiléen
Système {projectile}

2) $\downarrow \vec{g}$ - direction verticale
- sens vers le bas

dans le repère $(O, \vec{i}, \vec{j}, \vec{k})$ $\quad \vec{g} \begin{vmatrix} 0 \\ -g \\ 0 \end{vmatrix}$

3) $\vec{OM_0} \begin{vmatrix} x_0 = 0 \\ y_0 = h \\ z_0 = 0 \end{vmatrix}$

4) $\vec{v_0} \begin{vmatrix} v_{0x} = v_0 \cos\alpha \\ v_{0y} = v_0 \sin\alpha \\ v_{0z} = 0 \end{vmatrix}$

5) Bilan des forces : $\vec{P}$ (on néglige les frottements)

6) et 7)
$$\sum \vec{F}_{ext} = m\vec{a}$$
$$\vec{P} = m\vec{a}$$
$$\cancel{m}\vec{g} = \cancel{m}\vec{a}$$
$$\vec{g} = \vec{a}$$

$\vec{a}(t) \begin{vmatrix} a_x(t) = 0 \\ a_y(t) = -g \\ a_z(t) = 0 \end{vmatrix}$

C'est bien une chute libre puisqu'il n'y a que le poids

8)
$\vec{a}(t) \begin{vmatrix} a_x(t) = \dfrac{dv_x}{dt} = 0 \\ a_y(t) = \dfrac{dv_y}{dt} = -g \\ a_z(t) = \dfrac{dv_z}{dt} = 0 \end{vmatrix}$ $\quad \xRightarrow{\text{primitive}} \quad$ $\vec{v}(t) \begin{vmatrix} v_x(t) = C_1 \\ v_y(t) = -gt + C_2 \\ v_z(t) = C_3 \end{vmatrix}$

à $t = 0$

$\vec{v}(t=0) \begin{vmatrix} v_x(t=0) = C_1 = v_{0x} = v_0 \cos\alpha \\ v_y(t=0) = C_2 = v_{0y} = v_0 \sin\alpha \\ v_z(t=0) = C_3 = v_{0z} = 0 \end{vmatrix}$

d'où $\vec{v}(t) \begin{vmatrix} v_x(t) = v_0 \cos\alpha \\ v_y(t) = -gt + v_0 \sin\alpha \\ v_z(t) = 0 \end{vmatrix}$

9) on sait que $\vec{v} = \frac{d\vec{OM}}{dt}$

$$\vec{v}(t) \begin{cases} v_x(t) = \frac{dx}{dt} = v_0 \cos\alpha \\ v_y(t) = \frac{dy}{dt} = -gt + v_0 \sin\alpha \\ v_z(t) = \frac{dz}{dt} = 0 \end{cases} \xrightarrow{\text{primitive}} \vec{OM}(t) \begin{cases} x(t) = v_0 \cos\alpha\, t + C_4 \\ y(t) = -\frac{1}{2} g t^2 + v_0 \sin\alpha\, t + C_5 \\ z(t) = C_6 \end{cases}$$

$$\text{à } t=0 \quad \vec{OM}(t=0) \begin{cases} x(t=0) = C_4 = x_0 = 0 \\ y(t=0) = C_5 = y_0 = h \\ z(t=0) = C_6 = 0 \end{cases}$$

$$\text{d'où} \quad \vec{OM}(t) \begin{cases} x(t) = v_0 \cos\alpha\ t \\ y(t) = -\frac{1}{2} g t^2 + v_0 \sin\alpha\ t + h \\ \boxed{z(t) = 0} \end{cases}$$

→ mouvement plan

10) $x(t) = v_0 \cos\alpha\, t = 2{,}34\, t \Rightarrow v_0 \cos\alpha = 2{,}34 \text{ m s}^{-1}$

$y(t) = -\frac{1}{2} g t^2 + v_0 \sin\alpha\, t + h = -4{,}91 t^2 + 3{,}84\, t + 0{,}04$

$\Rightarrow v_0 \sin\alpha = 3{,}84 \text{ m s}^{-1}$

$$\frac{\cancel{v_0} \sin\alpha}{\cancel{v_0} \cos\alpha} = \frac{3{,}84}{2{,}34} = \tan\alpha$$

$$\alpha = \tan^{-1}\left(\frac{3{,}84}{2{,}34}\right) = 58{,}6°$$

$$v_0 \cos\alpha = 2{,}34 \Rightarrow v_0 = \frac{2{,}34}{\cos\alpha} = \frac{2{,}34}{\cos 58{,}6} = 4{,}5 \text{ m.s}^{-1}$$

11) $t = \frac{x}{v_0 \cos\alpha}$ $\quad y(x) = -\frac{1}{2} g \frac{x^2}{v_0^2 \cos^2\alpha} + \cancel{v_0} \sin\alpha \times \frac{x}{\cancel{v_0} \cos\alpha} + h$

$$y(x) = -\frac{g}{2 v_0^2 \cos^2\alpha} x^2 + \tan\alpha\, x + h$$

12) on doit résoudre $y(x) = H$

soit $-\frac{g}{2 v_0^2 \cos^2\alpha} x^2 + \tan\alpha\, x + (h - H) = 0$

$\Delta = 2{,}22 \Rightarrow x_1 = 0{,}25 \text{ m}$ et $x_2 = 1{,}42 \text{ m}$